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Probing Higgs Boson - and Supersymmetry-Induced CP Violation in Top Quark Production by (Un-) Polarized Electron-Positron Collisions

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Abstract:
We consider, both for unpolarized and longitudinally polarized electron beams, the reaction $\mathrm{e}^+\mathrm{e}^- \to \mathrm{t}\bar{\mathrm{t}}$ with subsequent semileptonic t and nonleptonic $\bar{t}$ decay and vice versa and investigate optimized angular correlations which are sensitive to CP non-conservation in the $t\bar{t}$ production vertex. We calculate these correlations for two-Higgs–doublet extensions and the minimal supersymmetric extension of the Standard Model (SM) with CP violation beyond the Kobayashi-Maskawa phase. While the sensitivity of the optimal correlation for tracing dispersive CP effects is enhanced with longitudinally polarized electron beams, we find that the sensitivity of the best correlation for probing absorptive CP effects is almost independent of the polarization degree.

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1 Introduction

One of the pillars of the physics program of a future high luminosity linear electron positron collider would be the detailed investigation of top quarks [1]. The production and decay of these quarks could be studied at such a facility under rather clean conditions. There are quite a number of proposals and detailed investigations how top quarks, and specifically associated polarization phenomena, can serve as probes of (non) Standard Model interactions [1] - [9]. In particular arguments have been given and proposals have been made to use top quarks produced in e^+e^- collisions as probes of CP-violating interactions ([7] - [18]) beyond the Kobayashi-Maskawa (KM) mechanism [19].

In this article we investigate, in continuation of [11], the effects on $\bar{t}t$ production of CP-violating interactions from an extended Higgs sector and from the minimal supersymmetric extension of the SM, propose observables for tracing these effects and calculate their expectation values. The new features of this article are: we take into account the possibility of longitudinally polarized electron beams which enhance some of the effects, and we propose and study optimized observables with maximized sensitivity to CP effects in the channels discussed below. (Some simple correlations were calculated in [16] for polarized e^- beams in terms of CP-violating dipole form factors.)

The paper is organized as follows: In section 2 we discuss how CP-violating effects in $e^+e^- \to \bar{t}t$ manifest themselves in top spin-momentum correlations and give observables to detect these correlations in "semihadronic" final states. The construction of the optimized observables is done in the appendix. In sections 3 and 4 we compute the expectation values of these observables and their sensitivities within the above-mentioned models. Section 5 contains our conclusions.

2 Observables

In the following we consider the production of a top quark pair via the collision of an unpolarized positron beam and a longitudinally polarized electron beam:

$$\mathrm{e}^+(\boldsymbol{e}_+) + \mathrm{e}^-(\boldsymbol{e}_-, p) \quad \to \quad \mathrm{t}(\boldsymbol{k}_\mathrm{t}) + \bar{\mathrm{t}}(\boldsymbol{k}_{\bar{\mathrm{t}}}). \tag{1}$$

Here p is the longitudinal polarization of the electron beam ($p = 1$ refers to right handed electrons). We are interested in reactions with semileptonic t decay and non-leptonic $\bar{\mathrm{t}}$ decay and vice versa:

$$\mathrm{t}\ \bar{\mathrm{t}} \quad \to \quad \ell^+(\boldsymbol{q}_+) + \nu_\ell + \mathrm{b} + \overline{X}_\mathrm{had}(\boldsymbol{q}_{\bar{X}}), \tag{2}$$

$$\mathrm{t}\ \bar{\mathrm{t}} \quad \to \quad X_\mathrm{had}(\boldsymbol{q}_X) + \ell^-(\boldsymbol{q}_-) + \bar{\nu}_\ell + \bar{\mathrm{b}}\ \ , \tag{3}$$

where the 3-momenta in eqs. (1) - (3) refer to the e^+e^- c. m. frame.
If non-standard CP-violating interactions exist they can affect the $\bar{t}t$ production and decay vertices. Quantum mechanical interference of the CP-even and -odd parts of the amplitudes for the above reactions then lead to the correlations which we are after. For the models of sect.3,4 below it has been shown [11] that they induce a CP-violating form factor in t (and $\bar{t}$) decay which is smaller than the electric and weak dipole form factors of the top generated in the production vertex. Therefore we consider below observables which are predominantly sensitive to these form factors.
The SM extensions of sect. 3,4 lead to electric and weak dipole form factors $d_t^{\gamma,Z}(s)$ which can have imaginary (i.e., absorptive) parts. The real parts $\mathrm{Re}d_t^{\gamma,Z}$ induce a difference in the t and $\bar{t}$ polarizations orthogonal to the scattering plane of reaction (1). Non-zero absorptive parts $\mathrm{Im}d_t^{\gamma,Z}$ lead to a difference in the t and $\bar{t}$ polarizations along the top direction of flight.
The class of events (2), (3) is highly suited to trace these spin-momentum correlations in the $\bar{t}t$ production vertex through final state momentum correlations: From the hadronic momentum in (2),(3) one can reconstruct the $\bar{\mathrm{t}}$ and t momentum, respectively and hence the rest frames of these quarks. Moreover the scattering plane of the reaction (1) can be determined in each event. The extremely short life time of the top quark implies that the top polarization is essentially undisturbed by hadronization effects and can be analyzed by its parity-violating weak decay $t \to b + W$. Further, the charged lepton from semileptonic top decay is known to be by far the best analyzer of the top spin [21].
Therefore the observables which will be discussed below are chosen to be functions of the directions of the hadronic system from top decay, of the charged lepton momentum, of the positron beam direction, and of the c.m. energy $\sqrt{s}$. In a previous article [11] we have used observables being functions of the lepton momenta in the laboratory system. Aiming at the optimization of those results we are lead to observables involving the lepton unit momenta $\hat{\boldsymbol{q}}^*_\pm$ in the corresponding top rest frames, which are directly accessible in the processes considered here.
The CP asymmetries which we discuss are differences of expectation values

$$\mathcal{A} \quad = \quad \langle \mathcal{O}_+(s, \hat{\boldsymbol{q}}^*_+, \hat{\boldsymbol{q}}_{\bar{X}}, \hat{\boldsymbol{e}}_+)\rangle - \langle \mathcal{O}_-(s, \hat{\boldsymbol{q}}^*_-, \hat{\boldsymbol{q}}_X, \hat{\boldsymbol{e}}_+)\rangle \quad , \tag{4}$$

where the mean values refer to events (2), (3) respectively. The observable $\mathcal{O}_-$ is defined to be the CP image of $\mathcal{O}_+$; that is, it is obtained from $\mathcal{O}_+$ by the substitutions $\hat{\boldsymbol{q}}_{\bar{X}} \to -\hat{\boldsymbol{q}}_X$, $\hat{\boldsymbol{q}}^*_+ \to -\hat{\boldsymbol{q}}^*_-$, $\hat{\boldsymbol{e}}_+ \to \hat{\boldsymbol{e}}_+$.
In the sections below we shall plot the ratios

$$r \quad = \quad \frac{\langle \mathcal{O}_+\rangle - \langle \mathcal{O}_-\rangle}{\Delta\mathcal{O}_+} \tag{5}$$

where $\Delta\mathcal{O} = \sqrt{\langle \mathcal{O}^2\rangle - \langle \mathcal{O}\rangle^2}$ is the width of the distribution of $\mathcal{O}$. For the observables used in this paper we have $\Delta\mathcal{O} \approx \sqrt{\langle \mathcal{O}^2\rangle}$ and $\Delta\mathcal{O}_+ \simeq \Delta\mathcal{O}_-$. The absolute value $|r|$ is a measure of the sensitivity of a correlation. The corresponding signal-to-noise ratio is given by $S_{\mathcal{A}} = |r|\sqrt{N}/\sqrt{2}$, where N is

the number of events of type (2) or (3).
In the appendix we derive explicit expressions for observables of the above type which lead to the largest possible sensitivity $|r|$. In addition to the functional dependence exhibited in (4) they depend also on the degree of beam polarization. At first sight these optimized observables may appear rather unhandy. Therefore we derive from the formulae given in the appendix two observables – which are sensitive to dispersive and absorptive CP effects, respectively — with a simpler structure:

$$\mathcal{O}_{+}^{\rm Re} \quad = \quad (\hat{\boldsymbol{q}}_{\bar{X}} \times \hat{\boldsymbol{q}}_{+}^{*}) \cdot \hat{\boldsymbol{e}}_{+} \quad , \tag{6}$$

$$\mathcal{O}_{+}^{\rm Im} \quad = \quad -[1 + (\frac{\sqrt{s}}{2m_t} - 1)(\hat{\boldsymbol{q}}_{\bar{X}} \cdot \hat{\boldsymbol{e}}_{+})^2]\hat{\boldsymbol{q}}_{+}^{*} \cdot \hat{\boldsymbol{q}}_{\bar{X}} + \frac{\sqrt{s}}{2m_t}\ \hat{\boldsymbol{q}}_{\bar{X}} \cdot \hat{\boldsymbol{e}}_{+}\ \hat{\boldsymbol{q}}_{+}^{*} \cdot \hat{\boldsymbol{e}}_{+} \quad , \tag{7}$$

where m_t denotes the top mass. The observables $\mathcal{O}_-$ are obtained from the corresponding $\mathcal{O}_+$ by the substitutions given below eq. (4). Eqs. (6),(7) do not depend on the beam polarization degree p.
In the narrow width approximation for $\bar{t}t$ production and decay it can be shown (see [7, 11]) that the resulting asymmetries $\mathcal{A}^{\rm Re}, \mathcal{A}^{\rm Im}$ are predominantly sensitive to CP violation in the $\bar{t}t$ production amplitude: $\mathcal{A}^{\rm Re}$ traces non-zero ${\rm Re}d_t^{\gamma,Z}$ whereas $\mathcal{A}^{\rm Im}$ receives contributions from the absorptive parts of these form factors.
From the structure of the observables (6),(7) we can read off whether the corresponding asymmetries depend on the polarization of the electron beam: Since $\mathcal{O}_{\pm}^{\rm Re}$ is linear in the beam direction $\hat{\boldsymbol{e}}_{+}$ its expectation value is proportional to the vector polarization of the intermediate virtual γ/Z boson which depends on p. We find that the corresponding ratio r is enhanced by a factor of about two if the electron beam is fully polarized as compared to the case $p = 0$ (see below). On the other hand $\mathcal{O}_{\pm}^{\rm Im}$ is bilinear in $\hat{\boldsymbol{e}}_{+}$, leading to an asymmetry which is blind to the beam polarization.
In the following we shall always consider phase space cuts which are CP-symmetric. When the e^+e^- beams are unpolarized (or transversely polarized) the asymmetries (4) can be classified as being odd under a CP transformation. This means that contributions to $\langle\mathcal{O}_{\pm}\rangle$ from CP–invariant interactions cancel in the difference. If the electron beam is longitudinally polarized the initial e^+e^- state is no longer CP-symmetric in its c.m. frame and the CP classification no longer applies. Contributions from CP-conserving interactions can, in principle, contaminate $\mathcal{A}$ if $p \neq 0$. However, in practice, this is not a problem for the following reasons: i) At a high luminosity linear collider CP-violating or -conserving interactions must induce ratios $|r| > 0.01$ in order to have a chance to be detectable. Effects in $\mathcal{A}$ at the per mill level are swamped by the statistical fluctuations (and probably overwhelmed by systematic errors in an experiment). ii) To leading order in the electroweak couplings the contamination problem does not arise for the reaction (1) since the intermediate virtual photon or Z boson is still a CP eigenstate in the c.m. frame for arbitrary polarization p. iii) The asymmetry $\mathcal{A}^{\rm Re}$ and the

corresponding optimized version of the appendix are T-odd, that is, odd under reversal of momenta. Hence only absorptive parts of the CP-conserving component of the amplitude, which arise at 1-loop order, can contribute. As to SM interactions only absorptive parts of electroweak box contributions to (1) are relevant in view of ii). They can be estimated to be of the order of $\pi \times e^2/(16\pi^2) \simeq 0.2\%$ and are therefore negligible. (It might nevertheless be interesting to investigate these contributions in detail.) iv) The asymmetry $\mathcal{A}^{\rm Im}$ and the corresponding optimized version of the appendix are T-even. For $p \neq 0$ the most important CP-conserving contribution to these quantities comes from helicity flip bremsstrahlung off the initial e^- or e^+. This effect has been calculated in [20] and was shown to be small. For our asymmetries it leads to a contamination of a few per mill at most.

3 Two–Higgs–doublet extensions of the Standard Model

Only new CP-violating interactions beyond the Kobayashi Maskawa phase can induce recognizable effects in asymmetries of the form (4). An intriguing interaction of this type is provided already by two–Higgs–doublet extensions of the SM with explicit CP violation in the scalar potential. These models contain in their spectrum three physical neutral Higgs bosons $\varphi_j (j = 1, 2, 3)$ which no longer have a definite CP parity. In these models the Yukawa couplings of the φ_j to the top quark are given by [10]:

$$\mathcal{L}_Y = -\left(\sqrt{2}G_F\right)^{\frac{1}{2}} \sum_{j=1}^{3} \left[a_{j\mathrm{t}} m_{\mathrm{t}} \bar{\mathrm{t}}\mathrm{t} + \tilde{a}_{j\mathrm{t}} m_t \bar{\mathrm{t}} i\gamma_5 \mathrm{t}\right] \varphi_j \tag{8}$$

where m_t is the mass of the top quark,

$$a_{j\mathrm{t}} = d_{2j}/\sin\beta, \quad \tilde{a}_{j\mathrm{t}} = -d_{3j}\cot\beta, \tag{9}$$

$\tan\beta = v_2/v_1$ is the ratio of vacuum expectation values of the two doublets, and d_{2j}, d_{3j} are the matrix elements of a 3×3 orthogonal matrix.
The Yukawa couplings (8) generate electric and weak dipole form factors of the top quark at one loop order which have absorptive parts for $s \geq 4m_t^2$. We shall assume that at least one of the φ's is light; for definiteness we take $m_{\varphi_1} << m_{\varphi_{2,3}}$. Heavy Higgs bosons lead to very small effects in the asymmetries of sect. 1 – contrary to the case of resonant φ_j production [27].) In the parameter range which we shall explore below the asymmetries (4) are, to a good approximation, proportional to

$$\gamma_{\rm CP} \quad \equiv \quad d_{21}d_{31}\cot\beta/\sin\beta. \tag{10}$$

One may define CP violation in the neutral Higgs sector to be maximal if $|d_{i1}| = 1/\sqrt{3}$ for i=1,2,3. K and B meson data indicate that $\tan\beta \gtrsim 0.3$. This translates into the rather loose upper bound: $|\gamma_{\rm CP}| \lesssim 4$. This bound is not in conflict with the present experimental upper bounds on the electric dipole

moment of the neutron [28] and of the electron [29]. Here we are interested in the case where the pseudoscalar coupling $\tilde{a}_{1\mathrm{t}}$ is not severely suppressed. If $\tan\beta >> 1$ then the effects, which we study in this paper, become too small for being observable. In the following we choose $\gamma_{\mathrm{CP}} = 2$, $m_{\mathrm{t}} = 180$ GeV, and $m_{\varphi_1} = 100$ GeV.
For these parameters the ratios r which correspond to the observables (6) and (7), respectively, are plotted in Figs. 1a,b as functions of the c.m. energy using the dipole form factors as obtained in [10]. From the figures one can infer the statistical sensitivities in straightforward fashion.
The asymmetry $\mathcal{A}^{\mathrm{Re}}$ is proportional to the real part of the dipole form factors. The corresponding ratio r has its maximum near the $t\bar{t}$ threshold, becomes zero around $\sqrt{s} \approx 500$ GeV (the zero originates from the real part of the form factors), and increases again. Beyond 1.2 TeV it eventually decreases. We find, as expected, a strong dependence of r on the longitudinal polarization of the electron beam: For $p = \pm 1$ the sensitivity is more than twice as large as in the unpolarized case.
The difference $\mathcal{A}^{\mathrm{Im}}$ projects onto the imaginary parts of the form factors. It leads to a ratio r which reaches its maximum of 2 percent at about 500 GeV and decreases moderately at higher energies. As mentioned above, the beam polarization has no effect on this asymmetry.
Instead of (6) and (7) we may use observables with optimized signal-to-noise ratio. This is done in appendix A. We find that the corresponding sensitivities increase by about 30 percent as compared to those given by Figs. 1a,b. Fig. 3b shows that the absorptive asymmetry (19) has the highest sensitivity to γ_{CP} and that it depends only weakly on the beam polarization. The maximal sensitivity is reached at $\sqrt{s} \simeq 450$ GeV where $|r_2| = 2.4\%$ (2%) if p=±1 (0). In order to detect this as a 3 s.d. effect one would need 31000 (45000) events of the type (2) and of (3). Since (2) and (3) correspond each to 2/9 of the $\bar{t}t$ events this would require about 4 years of data collection with a linear collider having a luminosity of $\mathcal{L} = 5 \times 10^{33}/cm^2s$. If the CP-violating effect is larger, say γ_{CP}=4, then only 1/4 of these events would be needed for a 3 s.d. effect.
In general the asymmetries become smaller with increasing Higgs mass. If we keep the CP-violating coupling $\gamma_{\mathrm{CP}} = 2$ but change the Higgs mass to m_φ = 200 GeV then the value of $|r|$ and $|r_1|$ of Fig.1a and Fig.3a, respectively, at $\sqrt{s} \approx 384$ GeV (which is the location of the maximal value close to threshold) and the maximal values of $|r|$ of Fig.1b and $|r_2|$ of Fig.3b at $\sqrt{s} \approx 450$ GeV are reduced by a factor of about 0.7.
In summary, for light Higgs masses m_φ <200 GeV and sizable CP-violating coupling γ_{CP} >2 there is a chance to see Higgs sector CP violation as a virtual effect in $\bar{t}t$ production. A light Higgs particle φ would also be produced at a linear collider. A consequence of φ not being CP eigenstate would be a CP violation effect in the φ fermion-antifermion amplitude at Born level which could be detected in $\varphi \to \tau^+\tau^-$ [22]. (For further checks of the CP properties of Higgs particles, see [23, 24].)

4 Minimal supersymmetric extension of the Standard Model

It is well known that in the minimal supersymmetric extension of the SM with two Higgs doublets CP-violating terms are absent in the Higgs potential. In the supersymmetric case neutral Higgs sector CP violation requires at least one additional scalar field, for instance a singlet. However, already in the minimal supersymmetric extension of the SM additional CP–violating phases (besides the KM phase) can be present in the Majorana mass terms, e.g. of the gluinos, and in the squark (and slepton) mass matrices. For mass eigenstates these phases then appear in the $\mathrm{t}\tilde{\mathrm{t}}$ – gluino couplings in the form (flavour mixing is ignored)

$$\mathcal{L}_{\tilde{\mathrm{t}}\mathrm{t}\lambda} = i\sqrt{2}\ g_{\mathrm{QCD}} \left\{ e^{i\phi_{\mathrm{t}}}\ \tilde{\mathrm{t}}_L^* T^a (\bar{\lambda}^a \mathrm{t}_L) + e^{-i\phi_{\mathrm{t}}}\ \tilde{\mathrm{t}}_R^* T^a (\bar{\lambda}^a \mathrm{t}_R) \right\} + h.c. \tag{11}$$

with $\phi_{\mathrm{t}} = \phi_\lambda - \phi_{\tilde{\mathrm{t}}}$ and

$$\tilde{\mathrm{t}}_L = \tilde{\mathrm{t}}_1 \cos\alpha_{\mathrm{t}} + \tilde{\mathrm{t}}_2 \sin\alpha_{\mathrm{t}}, \tag{12}$$

$$\tilde{\mathrm{t}}_R = -\tilde{\mathrm{t}}_1 \sin\alpha_{\mathrm{t}} + \tilde{\mathrm{t}}_2 \cos\alpha_{\mathrm{t}}, \tag{13}$$

where $\tilde{\mathrm{t}}_{1,2}$ denote the fields corresponding to mass eigenstates. If ϕ_{t} is "flavour-universal", i.e. is the same for the $\tilde{u}, \tilde{c}, \tilde{t}$ squarks, then the experimental upper bound on the electric dipole moment of the neutron puts a constraint on the magnitude of ϕ_{t} (see, e.g. the reviews [25, 26]), depending on the magnitude of the gluino and squark masses. If $m_\lambda, m_{\tilde{u}}, m_{\tilde{d}}$ are close to their present experimental lower bounds of about 200 GeV [30] (this bound holds only if $m_\lambda = m_{\tilde{q}}$) then $|\sin(\phi_{\mathrm{t}})| \leq 0.1$. However, for masses larger than 500 GeV this constraint disappears. In any case $\phi_{\tilde{\mathrm{t}}}$ may a priori be much larger in magnitude than $\phi_{\tilde{u}}$. In order to investigate the maximally possible magnitude of the effects we shall put $\sin(2\phi_{\mathrm{t}}) = 1$.
In order that the interaction (11) induces CP–violating dipole form factors in the $\mathrm{t}\bar{\mathrm{t}}$ production amplitude $\tilde{\mathrm{t}}_{1,2}$ must not be mass–degenerate. We use the form factor formulae of ([11]). In order to exhibit the typical size of the effects we assume maximal mixing, take the mass of $\tilde{\mathrm{t}}_2$ to be $m_2 = 400$ GeV and those of $\tilde{\mathrm{t}}_1$ in the vicinity of m_t, to wit: $m_1 = 190$ GeV. The gluino mass is chosen to be $m_\lambda = 150$ GeV which is in accord with [30]. For longitudinal electron polarization $p = 0, \pm 1$ the ratios r corresponding to the observables (6), (7) are plotted in Figs. 2a,b. As expected, the sensitivity of $\mathcal{A}^{\mathrm{Re}}$ is large around the threshold for $\tilde{\mathrm{t}}_1\tilde{\mathrm{t}}_1^*$ production. (In the close vicinity of a squark threshold our results may become unreliable due to resonance effects). Above a zero at about 750 GeV there is another local maximum of $|r| \approx= 0.6\%$ at $\sqrt{s} \approx 1.2$ TeV for p= -1, which is slightly larger than the maximum near threshold. The absorptive asymmetry $\mathcal{A}^{\mathrm{Im}}$ is largest in the region where the dispersive asymmetry is very small. For the parameters above the maximal sensitivity is 0.5 % at $\sqrt{s} \approx 600$ GeV independent of the polarization.
We have evaluated the ratios r also for other mixing angles, squark and

gluino masses, requiring that $m_\lambda, m_{1,2} \geq 150$ GeV. We have not found values of $|r|$ larger than the ones given above. (The location of the maxima do of course change.) If one uses the optimal observables (15) given in the appendix the corresponding sensitivities increase by about 30% as compared to those displayed in Figs. 2a,b. That is, even with these quantities we get ratios $|r| \lesssim 0.01$.

A number of supersymmetry induced CP correlations and asymmetries for $\bar{t}t$ production and decay were also studied in refs. [15, 17, 18]. The observables used in these references have a smaller sensitivity to supersymmetric CP violation than the ones which we have proposed and evaluated here.

5 Conclusions

We have constructed optimized observables to search for CP violation in $\bar{t}t$ production using "semihadronic" final states. We have taken into account the possibility of both unpolarized and longitudinally polarized electron beams, and have calculated the expectation values of these observables for Higgs boson-induced and supersymmetry-induced CP violation. We have found that the optimal correlation for tracing dispersive CP effects is enhanced with longitudinally polarized electron beams, whereas the best correlation for probing absorptive CP effects is almost independent of the polarization degree. Moreover we have shown that the the latter correlation has the highest sensitivity to CP-violating Higgs boson exchange.

CP phases in gluino exchange lead to ratios $|r| \lesssim 1\%$. These effects are too small to be detectable as a, say 3 s.d. effect at a linear collider with integrated luminosity of 50 (fb)$^{-1}$/year. On the other hand if a light Higgs boson with mass $m_\varphi < 200$ GeV and sizeable CP-violating couplings to top quarks exists then, as shown above, an effect could be seen with the absorptive observable proposed in this paper.

A Optimal observables

In this appendix we give explicit formulae for the optimized observables which trace effects of CP–violating dipole moments in the reactions (1), (2), (3). The method of optimizing the signal–to–noise ratio of observables has proven to be a powerful tool for CP studies in tau-pair production at LEP [31, 32]. In order to illustrate the procedure we consider a differential cross section which is of the schematic form $\mathrm{d}\sigma = \mathrm{d}\sigma_0 + \lambda \mathrm{d}\sigma_1$. Phase space variables which are not measured in a particular experiment are understood to be integrated out. The parameter λ is assumed to be small so that possible higher order terms in λ can be neglected. If one wants to measure λ by the mean value of an appropriate observable one can show [12] (see also [33]) that the optimal observable, that is, the one with the minimal statistical error is given by

$$\mathcal{O} = \mathrm{d}\sigma_1 / \mathrm{d}\sigma_0. \tag{14}$$

This observable remains optimal in the presence of phase space cuts. A generalization to the case of several parameters was given in [34]. In our case $\mathrm{d}\sigma_0$ denotes the CP–conserving part of the differential cross section of the above reactions, while $\lambda \mathrm{d}\sigma_1$ is the CP–violating contribution.
In our case the CP-violating term of the the differential cross section is linear in the real and the imaginary parts of the electric and the weak dipole form factor of the top quark; i.e., it is of the form $\sum \lambda_i \mathrm{d}\sigma_1^i$. Each of these four couplings, which are a priori unknown, can be measured with observables $\mathcal{O}(i) = \mathrm{d}\sigma_1^i/\mathrm{d}\sigma_0$.
For practical purposes it is sufficient to use the tree level approximation of the transition matrix element squared when writing down the observables $\mathcal{O}(i)$. We give expressions for the final states (2) and (3) under the proviso that, for given c.m. energy (and top mass), the $\mathcal{O}(i)$ depend only on the the momentum direction $\hat{\boldsymbol{q}}_+^*$ of the charged lepton in the rest system of the top quark and on the $\bar{t}$ momentum direction in the laboratory frame when considering the channels (2), and analogously for the final states (3). Then the analytic form of the optimized observables is not too complicated; partly because in semileptonic top decays the energy and the direction of flight of the lepton are not correlated to lowest order, and because the terms which involve correlations between the lepton momenta $\hat{\boldsymbol{q}}_+^*$ and $\hat{\boldsymbol{q}}_-^*$ originating from $t\bar{t}$ spin correlations do not contribute. The optimized observables have the following form (as before the labels $\pm$ refer to the final states (2) and (3), respectively):

$$\mathcal{O}_\pm(i) = \frac{\displaystyle\sum_{B_1,B_2=\gamma,Z} \frac{R^1_\pm(B_1,B_2)_i}{(s-M^2_{B_1})(s-M^2_{B_2})}}{\displaystyle\sum_{B_1,B_2=\gamma,Z} \frac{R^0_\pm(B_1,B_2)}{(s-M^2_{B_1})(s-M^2_{B_2})}} \quad . \tag{15}$$

The $R^0_\pm$ are proportional to the respective Standard Model Born matrix elements. (Overall multiplicative constants or s-dependent terms are irrelevant and can be neglected.) For R^0_+ we have:

$$\begin{aligned}
R^0_+(B_1,B_2) \quad &= \\
&\Big(v_e^{B_1}(v_e^{B_2}-pa_e^{B_2}) + a_e^{B_1}(a_e^{B_2}-pv_e^{B_2})\Big) \\
&\quad \cdot\Big[v_t^{B_1}v_t^{B_2}(k_0^2+m_t^2+k^2(\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{e}}_+)^2) + a_t^{B_1}a_t^{B_2}k^2(1+(\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{e}}_+)^2) \\
&\quad +(v_t^{B_1}a_t^{B_2}+a_t^{B_1}v_t^{B_2})k(\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{q}}_+^*(k_0+(k_0-m_t)(\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{e}}_+)^2) + m_t\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{e}}_+\ \ \hat{\boldsymbol{e}}_+\cdot\hat{\boldsymbol{q}}_+^*)\Big] \\
+&\Big(v_e^{B_1}(a_e^{B_2}-pv_e^{B_2}) + a_e^{B_1}(v_e^{B_2}-pa_e^{B_2})\Big) \\
&\quad \cdot\Big[v_t^{B_1}v_t^{B_2}2k_0(m_t\hat{\boldsymbol{e}}_+\cdot\hat{\boldsymbol{q}}_+^* + (k_0-m_t)\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{e}}_+\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{q}}_+^*) \\
&\quad +a_t^{B_1}a_t^{B_2}2k^2\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{e}}_+\ \ \hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{q}}_+^* \quad + (v_t^{B_1}a_t^{B_2}+a_t^{B_1}v_t^{B_2})2k_0k\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{e}}_+\Big]
\end{aligned} \tag{16}$$

Here $\hat{\boldsymbol{k}}$ denotes the direction of the antitop quark (to be identified with $\hat{\boldsymbol{q}}_{\bar{X}}$) in the laboratory frame, $k = \sqrt{s/4-m_t^2}$, and $k_0 = \sqrt{s}/2$. The neutral electroweak couplings of $f = e, t$ are given by

$$v_f^\gamma = Q_f \cdot e \qquad\qquad v_f^Z = \frac{T_f^3 - 2Q_f \sin^2\theta_W}{2\sin\theta_W\cos\theta_W}\cdot e \tag{17}$$

$$a_f^\gamma = 0 \qquad\qquad a_f^Z = \frac{T_f^3}{2\sin\theta_W\cos\theta_W}\cdot e$$

The expression for R_-^0 is obtained from eq.(16) by the substitution $\hat{\boldsymbol{q}}_+^* \to -\hat{\boldsymbol{q}}_-^*$, and in experimental applications one should also replace the $\bar{t}$ unit momentum $\hat{\boldsymbol{k}}$ by $-\hat{\boldsymbol{k}}_{top} = -\hat{\boldsymbol{q}}_X$.
The CP-violating part of the respective differential cross section depends on the real and imaginary parts of the electric and weak dipole form factors. CP-violating form factors in the t and $\bar{t}$ decay vertices play no role in our case, as shown in [7, 11]. For the final state (2) the CP-violating matrix element squared is proportional to

$$\begin{aligned}
R_+^1(B_1,B_2) \quad &= \\
&\left(v_e^{B_1}(a_e^{B_2}-pv_e^{B_2})+a_e^{B_1}(v_e^{B_2}-pa_e^{B_2})\right)\left(v_t^{B_1}\mathrm{Re}d_t^{B_2}+v_t^{B_2}\mathrm{Re}d_t^{B_1}\right) \\
&\quad\cdot k_0^2 k(\hat{\boldsymbol{k}}\times\hat{\boldsymbol{q}}_+^*)\cdot\hat{\boldsymbol{e}}_+ \\
&+\left(v_e^{B_1}(v_e^{B_2}-pa_e^{B_2})+a_e^{B_1}(a_e^{B_2}-pv_e^{B_2})\right)\left(a_t^{B_1}\mathrm{Re}d_t^{B_2}+a_t^{B_2}\mathrm{Re}d_t^{B_1}\right) \\
&\quad\cdot k_0 k^2\ \hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{e}}_+\ (\hat{\boldsymbol{k}}\times\hat{\boldsymbol{q}}_+^*)\cdot\hat{\boldsymbol{e}}_+ \\
&+\left(v_e^{B_1}(v_e^{B_2}-pa_e^{B_2})+a_e^{B_1}(a_e^{B_2}-pv_e^{B_2})\right)\left(v_t^{B_1}\mathrm{Im}d_t^{B_2}+v_t^{B_2}\mathrm{Im}d_t^{B_1}\right) \\
&\quad\cdot k_0 k\left[-(m_t+(k_0-m_t)(\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{e}}_+)^2)\hat{\boldsymbol{q}}_+^*\cdot\hat{\boldsymbol{k}}+k_0\ \hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{e}}_+\ \hat{\boldsymbol{q}}_+^*\cdot\hat{\boldsymbol{e}}_+\right] \\
&+\left(v_e^{B_1}(a_e^{B_2}-pv_e^{B_2})+a_e^{B_1}(v_e^{B_2}-pa_e^{B_2})\right)\left(a_t^{B_1}\mathrm{Im}d_t^{B_2}+a_t^{B_2}\mathrm{Im}d_t^{B_1}\right) \\
&\quad\cdot k_0 k^2\left[\hat{\boldsymbol{q}}_+^*\cdot\hat{\boldsymbol{e}}_+-\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{e}}_+\ \hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{q}}_+^*\right] \quad .
\end{aligned} \tag{18}$$

The expression for R_-^1 is obtained from (18) by the substitutions given below eq.(17). The coefficients of $\mathrm{Re}d_t^{\gamma,Z}$ and $\mathrm{Im}d_t^{\gamma,Z}$ in $R_\pm^1$ then define the $R_{\pm i}^1$ which appear in (15). Note that the $\mathcal{O}(i)$ are functions of the polarization degree p of the electron beam.
In the threshold domain the terms proportional to the vector couplings v_t are the dominant ones in $R_\pm^1$, while the axial couplings a_t of the top quark come with an extra p–wave suppression factor k. Keeping only the terms which are dominant – as long as one is not too far above threshold – one is led to the observables (6),(7) which do not depend on p.
In the following we concentrate on the Higgs models of sect.3 as we have seen that this type of CP-nonconservation can give larger effects than the SUSY phases discussed in sect.4. For the models of sect.3 one has $d_t^\gamma(s) \approx 3d_t^Z(s)$ in the parameter range of interest. Inserting this relation into eq.(18) we can then write down two optimized observables which we denote by $\mathcal{O}_\pm(1)$ and $\mathcal{O}_\pm(2)$. They trace dispersive and absorptive CP effects, respectively. In Figs. 3a,b we have plotted the ratios

$$r_i \quad = \quad \frac{\langle\mathcal{O}_+(i)\rangle - \langle\mathcal{O}_-(i)\rangle}{\Delta\mathcal{O}_+(i)} \quad (i=1,2), \tag{19}$$

as a function of the c.m. energy for polarizations $p = 0, \pm 1$. Comparing with Figs. 1a,b one sees that at the energies where the respective sensitivities are highest one gains in sensitivity by about 30 % when using the optimal observables rather than those of eqs. (6), (7). Moreover, since the dominant terms in $\mathcal{O}(1)$ are linear in the beam direction $\hat{\boldsymbol{e}}_+$ whereas the dominant terms in $\mathcal{O}(2)$ are bilinear in $\hat{\boldsymbol{e}}_+$, it is clear that r_1 depends stronger on p than r_2.
Figs. 3a,b show that, when the e^- beam is maximally polarized, the sensitivity of the optimized observables is the same for $p = 1$ and $p = -1$. This result can be derived analytically for the ratios $r_{1,2}$ by straightforward algebra. The essential point is as follows: Consider the quantity

$$C_p(B_1, B_2) \quad = \quad \frac{v_e^{B_1}(a_e^{B_2} - p v_e^{B_2}) + a_e^{B_1}(v_e^{B_2} - p a_e^{B_2})}{v_e^{B_1}(v_e^{B_2} - p a_e^{B_2}) + a_e^{B_1}(a_e^{B_2} - p v_e^{B_2})} \quad , \tag{20}$$

where $B_1, B_2 = \gamma, \mathrm{Z}$. For $B_1 = B_2$ (20) can be interpreted as the vector polarization of the virtual γ/Z–intermediate state. It appears both in $R^0_\pm$ and $R^1_\pm$ when factoring out the term which forms the denominator of eq.(20). If $p = \pm 1$ then C_p does not depend on coupling constants:

$$C_{p=\pm 1}(B_1, B_2) \quad = \quad \mp 1 \quad . \tag{21}$$

The ratios $r_{1,2}$ can be shown to be even functions of $C_p(B_1, B_2)$ and therefore do not discriminate between $p = 1$ and $p = -1$. Of course if $|p| \neq 1$ these ratios do depend on the sign of p.

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Figure Captions

Fig 1a: Ratio r for observable $\mathcal{O}_{\pm}^{\rm Re}$ evaluated with parameters $m_{\rm t} = 180$ GeV, $m_{\varphi_1} = 100$ GeV, and $\gamma_{\rm CP} = 2$. The longitudinal electron polarization is $p = 0, \pm 1$.

Fig 1b: Ratio r for observable $\mathcal{O}_{\pm}^{\rm Im}$ with parameters as in Fig. 1a.

Fig 2a: Ratio r for observable $\mathcal{O}_{\pm}^{\rm Re}$ evaluated with parameters $\sin 2\phi_{\rm t} = 1$, $m_{\rm t} = 180$ GeV, $m_\lambda = 150$ GeV, $m_1 = 190$ GeV, $m_2 = 400$ GeV, and $\alpha_{\rm t} = \pi/4$ for electron polarization $p = 0, \pm 1$.

Fig 2b: Ratio r for observable $\mathcal{O}_{\pm}^{\rm Im}$ with parameters as in Fig 2a.

Fig 3a: Ratio r_1 defined in eq.(19) evaluated in the Higgs model with parameters as in Fig. 1a.

Fig 3b: Ratio r_2 defined in eq.(19) evaluated in the Higgs model with parameters as in Fig. 1a.